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The Norwegian Height System NN1954 Revisited

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Abstract. *Vertical reference systems based on national precise levelling networks have been realised in Europe since the 1860's. The heights are related to gravity, which by convention may be measured values or a model. The Norwegian Height System was adjusted in 1956 when the southern part of the country had been levelled once. Because of the lack of measured gravity, Clairaut's formula for gravity was used in the orthometric correction. The intention was to establish an orthometric height system, but this was never tested or confirmed. In the literature the system is often referred to as orthometric, but sometimes as normal orthometric. This study shows that the Norwegian Height System was realised by a strongly deformed net due to lack of land uplift corrections. The derived heights are neither normal nor orthometric. When the land uplift is handled in a proper way, the heights are close to normal heights. The remaining small differences are shown to depend on the free air anomaly.*

Key words. *NN1954, normal height, spheroidal-orthometric height, orthometric height, postglacial rebound*

1 Introduction

Early attempts to establish a height system for Norway were rooted in the 1864 general assembly of the Mittel-Europäische Gradmessung in Berlin. A tide gauge was mounted in the harbour of Oslo in 1876 to provide a reference level by averaging sea level recordings over several years. Levelling began in 1877, but progress was slow. In 1890 a reference marker for levelling was established on the property of the Geographical Survey of Norway in Oslo and its position relative to the tide gauge was determined.

In 1916 modern levelling instruments were acquired and the observing program gained momentum. Measurements were made along primary communication lines south of the Arctic Circle, but the calendar would show 1953 before the southern part of the country (between latitude 58.0°and 66.3°) was covered. In the meantime, tide gauges had been set up at several locations along the coast. The time series revealed a long term slope due to postglacial uplift, but the slope value varied from site to site. In Oslo it was found to be about 3 mm per year. Oslo was thus considered an unpractical site for a reference marker, and a new one was established at Tregde (near the south tip of Norway) where the measured uplift is very close to zero. The actual determination of apparent mean sea level was based on an average of seven tide gauges along the coast.

The entire data set was adjusted in 1956 and the result reported in (Trovaag and Jelstrup, 1956). The height system is called NN1954. A fundamental problem arises because the observing program needed almost 40 years to be completed in a region where postglacial rebound effects had been known to exist for a long time. No reliable model existed, however, so the adjustment procedure did not take land uplift into account at all. An additional problem was caused by the lack of observed gravity values along the levelling lines and loops. Thus Clairaut's formula

$$\gamma_{Clairaut} = \gamma_0(1 - \alpha\cos 2\phi - \beta h) \tag{1}$$

was used to obtain normal gravity values for a chosen reference Earth spheroid. Here is $\alpha = 2.6444 \times 10^{-3}$, $\beta = 0.314 \times 10^{-6} m^{-1}$, h is in meters and γ_0 is normal gravity for latitude 45°. The gravity difference between two nearby points at equal height is obtained by differentiating Clairaut's formula,

$$\Delta\gamma_{Clairaut} = \gamma_0 2\alpha \sin 2\phi\, \Delta\phi, \tag{2}$$

which is valid when $\Delta\phi$ is small. Referring to Fig. 1, the geopotential numbers at A and B are interrelated by

$$H_B\bar{g}_2 = H_A\bar{g}_1 + (\sum \Delta n)g_m, \tag{3}$$

where $\bar{g}_1$ and $\bar{g}_2$ are mean gravity values along the plumblines of A and B, respectively. For short distances AB the gravity for the whole line can be approximated by the average of the end points,

$$g_m = \frac{\bar{g}_1 + \bar{g}_2}{2}$$

Introducing $\Delta g = \bar{g}_2 - \bar{g}_1$, we obtain

$$H_B = H_A + \sum \Delta n - \frac{\Delta g}{\bar{g}_2}\left(H_A + \frac{\sum \Delta n}{2}\right). \tag{4}$$

The parenthesis represents the average height, h, between A and B and the last term may be written as $\frac{\Delta g}{\bar{g}_2}h$. Since no observed gravity values were available,

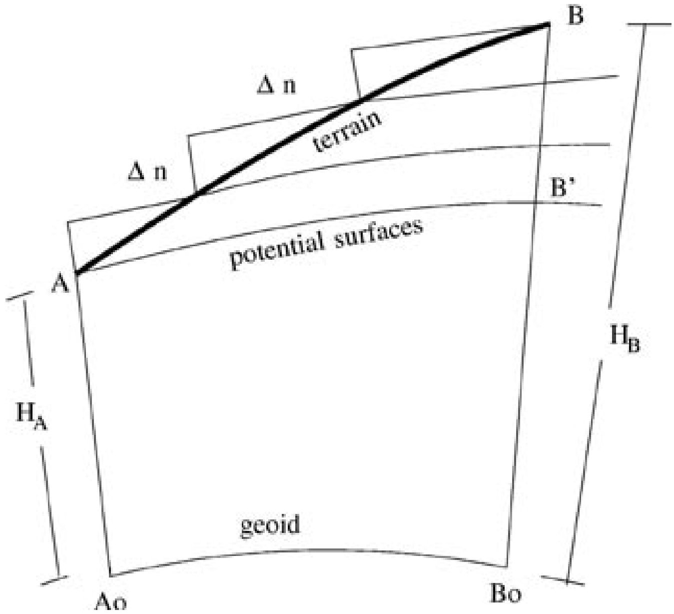

Figure 1. *The relationship between the orthometric heights and gravity (i.e. geopotential numbers) in two points on a levelling line.*

they were replaced by the corresponding normal values. Arguing that this last term is small, $\bar{g}_2$ was replaced by γ_0 to obtain a spheroidal-orthometric correction of

$$corr = \frac{\Delta\gamma_{Clairaut}}{\gamma_0} h = 2\alpha \sin 2\phi \, \Delta\phi \, h. \tag{5}$$

The levelling net north of the Arctic Circle was originally referred to the tide gauge at Narvik and was thus a separate entity. It was combined with the net south of the Arctic Circle in 1974 to generate a common height system for mainland Norway. The analysis in this paper considers the region south of the Arctic Circle exclusively, and does not address the attachment and combination of the northern branch.

2 Data and Methodology

This study aims to determine whether the original realisation of the Norwegian Height System yields normal heights or orthometric heights. This is done by comparing the official height system with theoretically "ideal" calculated normal heights and orthometric heights.

2.1 Heights

Precise height determination in wide areas must apply geopotential numbers, since levelling alone does not yield unambiguous height values. Levelling is combined with gravity, and heights are derived from the formula, (Heiskanen and Moritz, 1967, page 171):

$$height = \frac{C}{G}, \tag{6}$$

where C is geopotential number and G is gravity. The type of height obtained depends on the choice of gravity G. If mean gravity along the plumb line is used, orthometric heights are achieved. The use of mean normal gravity $\bar{\gamma}$, however, yields normal heights. In some regions, such as Norway, postglacial rebound must be considered as well in order to achieve accurate height values.

Historically, geopotential numbers were not used directly. Corrections derived from geopotential numbers were applied to the levelling values instead, (Heiskanen and Moritz, 1967, chap. 4). Unambiguous heights were achieved by,

$$height_2 = height_1 + \Delta n + corr. \tag{7}$$

This last approach was used when NN1954 was established. The corrections were computed according to Eq. (5). The type of height depends on the type of correction.

In the present investigations "ideal" normal heights and orthometric heights are obtained by adjusting geopotential numbers corrected for land uplift, and by calculation of mean normal gravity values $\bar{\gamma}$, and mean gravity values $\bar{g}$ along the plumb line, respectively. The mean normal gravity is given by:

$$\bar{\gamma} = \gamma\left(1 - (1 + f + m - 2f\sin^2\phi)\frac{H^*}{a} + \frac{H^{*2}}{a^2}\right) \tag{8}$$

where f is the flattening, a is the semi major axis of the GRS80 ellipsoid, γ is normal gravity at the ellipsoid, m is the Clairaut's constant, H^* is the normal height and ϕ is the geodetic latitude, (Heiskanen and Moritz, 1967, p. 170). The mean gravity is approximated by:

$$\bar{g} = g + 0.0424H \tag{9}$$

where g is measured gravity at the Earth's surface and H is orthometric height. The real mean gravity cannot be found since we do not know the density of the Earth, and cannot measure gravity inside the Earth's crust. (More details in (Heiskanen and Moritz, 1967).) Both mean normal gravity and mean gravity are calculated in an iterative process, since they depend on the height.

2.2 The official height system database

The official height database of the Norwegian Mapping Authority contains height values and coordinates in the EUREF89 frame for all geodetic markers ever included in precise levelling in Norway. All original levelling measurements since 1916 are listed, with corresponding spheroidal-orthometric corrections and gravity values for each marker, either measured or computed from existing nearby gravity observations. An algorithm allows geopotential numbers and height differences between any two markers to be generated, with or without spheroidal-orthometric corrections and land uplift corrections according to a model by (Danielsen, 2001).

2.3 The uplift model of Danielsen

The land uplift model of (Danielsen, 2001) covers Fennoscandia. It is based on first order levelling made by the Finnish Geodetic Institute, the National Land Survey in Sweden, the Norwegian Mapping Authority (SK) and the Norwegian State Railways, repeated GPS measurements and many of the mareographs in the area. Two methods, least squares collocation and "geometric filter", are used to determine the uplift rates. They produce separately images of the land uplift which are almost identical, and with quite similar accuracy. The model used here is based on least squares collocation. The formal accuracy is about 0.5 mm/year and varies slightly throughout the country (Danielsen, 2001).

3 Calculation and results

3.1 The Norwegian Height System

In order to check our adjustment procedure we first reconstructed the official height system using the original height differences with spheroidal-orthometric corrections added. The overall agreement was better than 3 mm.

Next we adjusted a set of geopotential numbers for which the empirical uplift model of Danielsen had been applied. The geopotential numbers are thus referred to the same epoch, 1954. The reference marker at Tregde is kept fixed, as in the original adjustment. "Ideal" orthometric heights and "ideal" normal heights were then calculated as described above, Eq. (6). The differences relative to the official heights are shown in Fig. 2 and 3.

Fig. 2 and 3 show small differences in the south, but generally the differences are large. Most markers differ by more than 8 cm. In Fig. 2 the differences appear to increase with altitude and also in general towards north-east. This latter effect may also be noted in Fig. 3, but to a lesser extent. The most noticeable differences are however, located along a section of the Norwegian-Swedish border. In the same region the uplift model of Danielsen shows the largest uplift values. NN1954 is a strongly deformed net, and it is impossible to conclude whether the official heights are orthometric heights or normal heights.

The date of observation explains some of the deformation. The first levelling reached the border in the 1930's. An uplift of 4–5 mm/year in the area between latitude 60° and 63° produces a mismatch of approximately 9 cm in 2 decades (1954–1930). Thus the differences are 9 cm too large here. This is especially true near Røros. Observations from 1918 and an uplift of 4–5 mm/year produces an uplift correction of 15 cm (1954–1918).

More recent measurements have not been corrected for land uplift either. The errors caused by the omission of the uplift corrections in the 1956 adjustment prop-

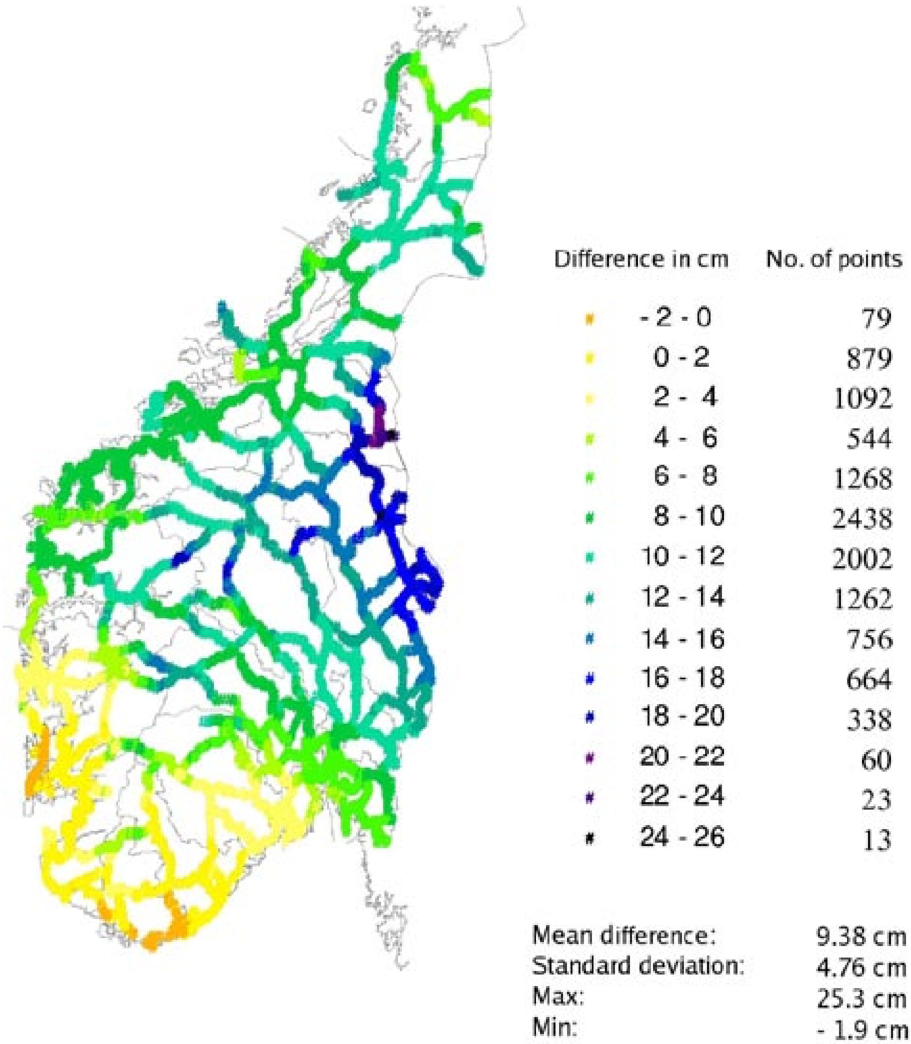

Figure 2. *Orthometric heights minus NN1954 heights.*

agates into the expansions of the net. Some levelling lines are pure expansions, and corrections to these lead to quite small differences. The levelling line located inside circle A in Fig. 3 and 5 is an example of this. This line was measured in 1983, and with a land uplift of approximately 3.5 mm/year the correction would be 10 cm if we refer back to 1954. The differences in Fig. 3 are for this line between 10 cm and 12 cm. When the uplift effect is corrected for in the height system, the differences are reduced to 0–2 cm (see Fig. 5).

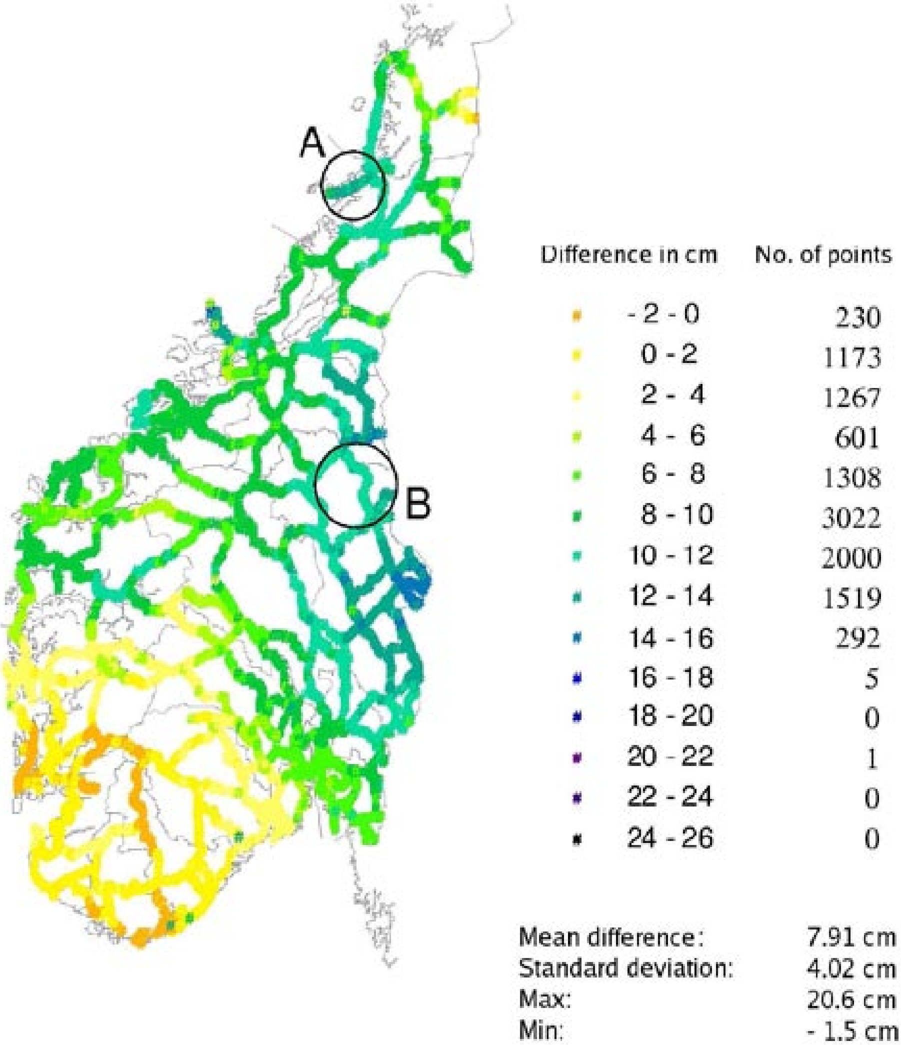

Figure 3. *Normal heights minus NN1954 heights.*

Other levelling lines are forced into the existing net and a correction here would only take away some part of the deformation. The levelling line inside circle B in Fig. 3 and 5 is such a line. In the north the line is attached to a point measured in 1918, and in the south to a point measured for the first time in 1935.

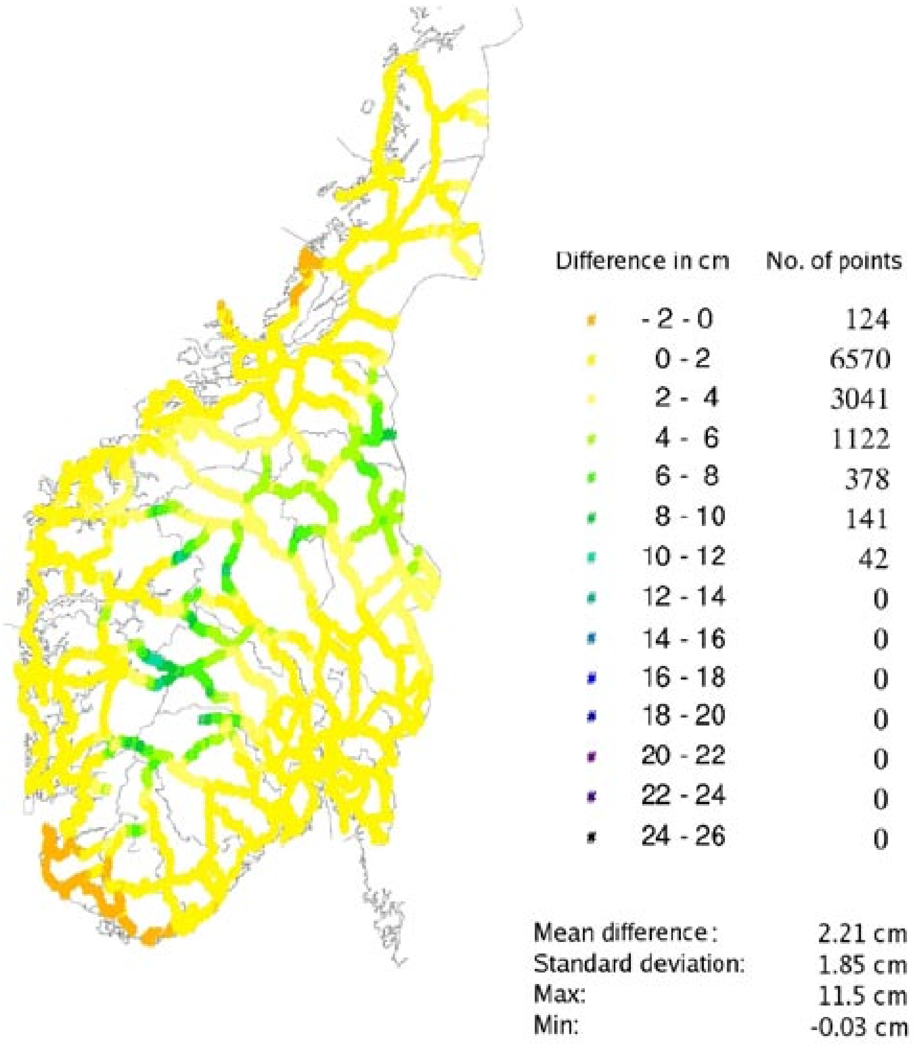

Figure 4. *Orthometric heights minus NN1954 heights corrected for land uplift.*

3.2 Corrections for land uplift

The effect of postglacial rebound on the NN1954-data is significant. The database entries are thus corrected by applying the empirical uplift model of (Danielsen, 2001). Only then it is hoped to decide if the use of Clairaut's gravity instead of measured gravity yields normal heights or orthometric heights. Therefore the official height system was recalculated by referring to one joint epoch, 1954.

A third dataset from the database was used for this purpose. Spheroidal-

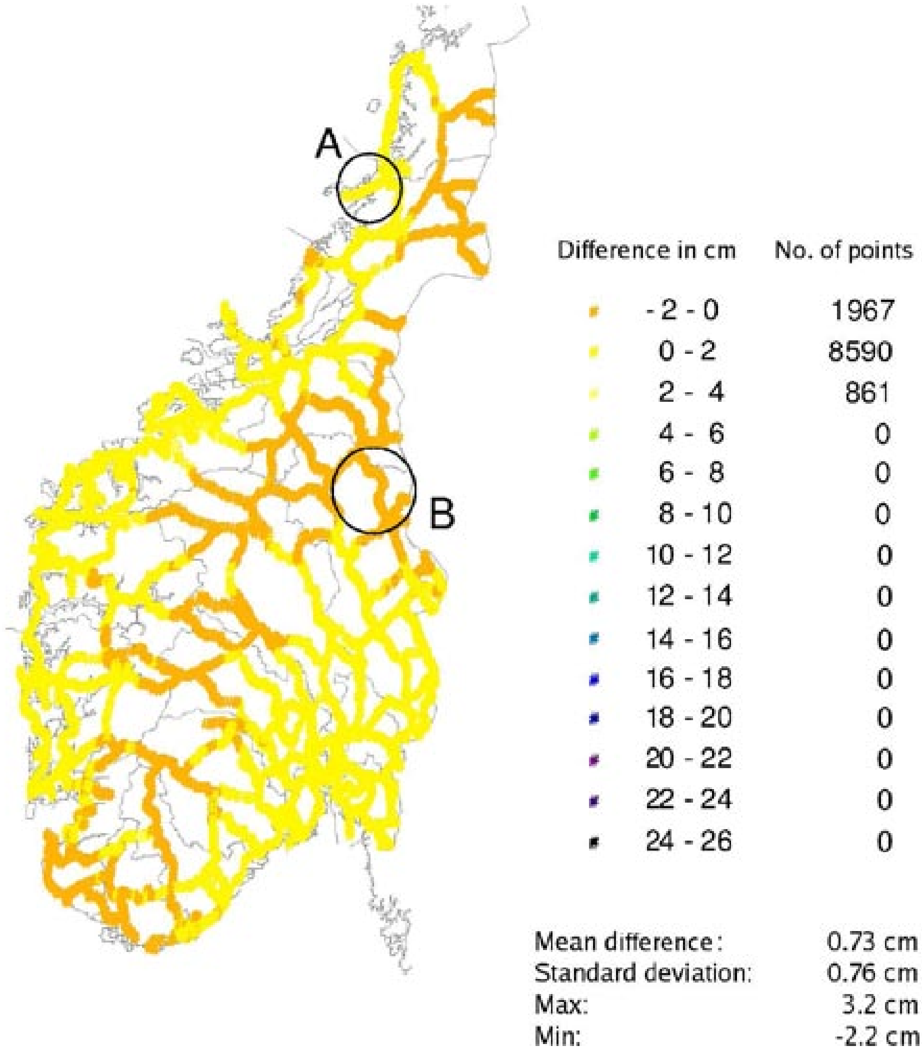

Figure 5. *Normal heights minus NN1954 heights corrected for land uplift.*

orthometric corrections and the empirical uplift model of (Danielsen, 2001) were applied to the height differences. The set was adjusted and compared with "ideal" orthometric heights and "ideal" normal heights. The results are shown in Fig. 4 and 5.

Now the large differences along the Swedish border diminish in both figures. This demonstrates that almost all deformation shown in Fig. 3 and 2 was due

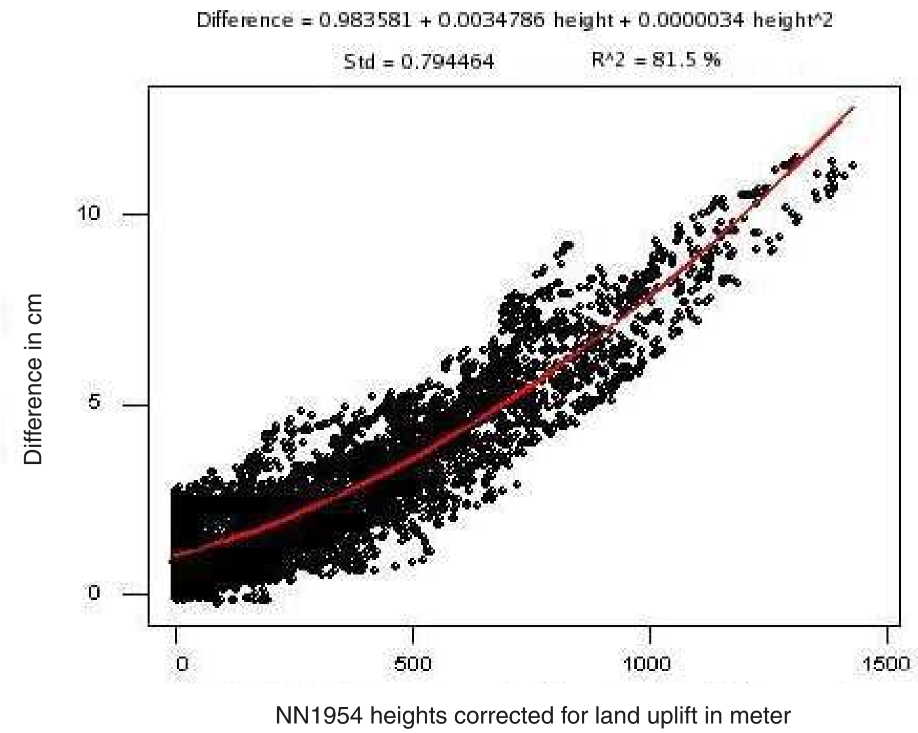

Figure 6. *Orthometric heights minus NN1954 heights corrected for land uplift, plotted against height. A second order polynomial has been fitted to the data.*

to the postglacial rebound. In Fig. 4 the differences increase with altitude. The NN1954 heights corrected for uplift are clearly too small in the mountainous areas compared to orthometric heights.

(Heiskanen and Moritz, 1967) chapter 8–13 expresses the difference between normal heights and orthometric heights by the height and the Bouguer anomaly.

$$H^* - H \approx \frac{\Delta g_B}{\bar{\gamma}} H. \tag{10}$$

The Bouguer anomaly, Δg_B, depends on the height as well, so the difference between normal heights and orthometric heights should vary quadratically with height. This is confirmed to hold for the Norwegian Height System (cf. Fig. 6). By fitting a quadratic function we obtain a correlation of 0.9.

Fig. 5 shows differences in the interval -2–3 cm, but most points have differences less than 1 cm. The largest difference in Fig. 3 is 17 cm and the difference in that point reduces to 0.6 cm in Fig. 5 when uplift is corrected for. The mean difference reduces from 8 cm to 0.7 cm and the standard deviation from 4 cm to 0.8 cm.

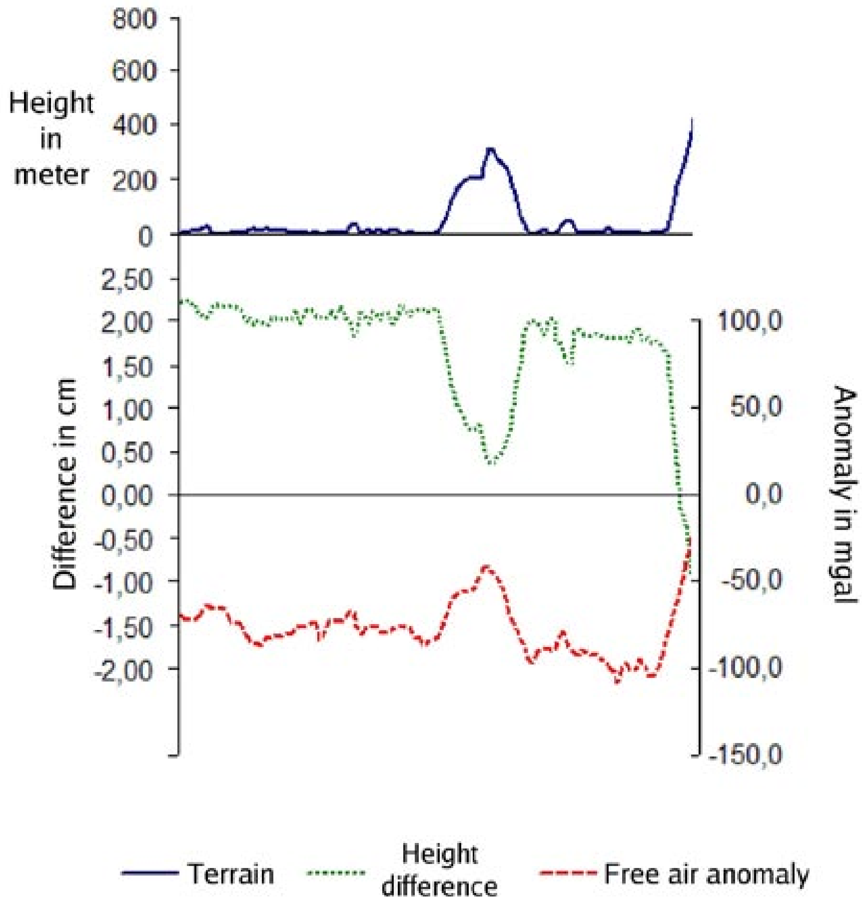

Figure 7. *Height difference's (i.e. normal heights minus NN1954 heights corrected for land uplift) dependency on free air anomaly.*

Applying the uplift model of (Danielsen, 2001) really improves the height system and yields a more homogeneous net. The quadratic dependence and the small differences in Fig. 5 argues that the use of Clairaut's computed gravity yields heights closest to normal heights.

There are some small differences remaining in Fig. 5. The only distinction between normal heights and NN1954 heights corrected for land uplift is the type of gravity used in the computation. Measured gravity values are used along the levelling lines to obtain normal heights. The uplift corrected NN1954 heights, however, use Clairaut's gravity. Clairaut's gravity and normal gravity differ by no more than 2 mGal within the test area of this study. Eq. (1) is thus an early formula to compute normal gravity at a certain altitude. The difference between measured gravity and normal gravity is the free air anomaly. The variation of the remaining height differences in Fig. 5 thus depends on the free air anomaly.

Fig. 7 shows this dependency. The free air anomaly (red line) is negative in this case, thus measured gravity is less than the normal gravity. If the levelling line is followed not much happens with the height differences (green line) as long as the terrain (blue line) is not changing much. Once the terrain starts to climb gravity decreases. The normal gravity decreases more than measured gravity because of the attraction of the topographic masses. Thus the free air anomaly increases. The normal heights lose relative to NN1954 heights corrected for land uplift and continue to lose as long as the altitude increases. When the terrain descends, the opposite happens. The free air anomaly decreases because normal gravity decreases more than the measured gravity. Normal heights gain relative to NN1954 heights corrected for land uplift and the height difference increases.

4 Conclusion

The Norwegian Height System yields neither orthometric nor normal heights. It is a strongly deformed net which should be recalculated in order to yield reliable height values. Compared to orthometric heights, NN1954-heights are generally too low in the mountainous regions. The fit is better in the lowland, but there are some areas with differences much larger than average. NN1954 heights are throughout the country closer to normal heights, especially in the mountains, but large differences appear here as well.

When the net is corrected for the land uplift, the heights remain clearly too low in the mountainous areas compared to orthometric heights, while they fit well to normal heights over the whole region. Fig. 4 is a typical picture of differences between orthometric heights and normal heights. Since we are comparing the recalculated net with orthometric heights this reveals the recalculated heights to be approximately normal heights. The quadratic dependence in Fig. 6 does also show this.

It is clear that the method used when NN1954 was established yields results very close to normal heights, (provided small land uplift and modest free air anomalies). The remaining differences are due to the difference between observed gravity and normal gravity, the free air anomaly.

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