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## The Local Geoid Model of Cameroon: CGM05

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**Abstract:** *This paper deals with the geoid determination in Cameroon by a gravimetric solution. A number of data files were compiled for this work, containing about 62,000 points on land and ocean areas and also including data derived from satellite altimetry. A hybrid global geopotential model (EGM-GGM) supplied the longer wavelength components of this geoid model, CGM05. This global model is obtained by adjusting the GRACE model GGM02C to degree and order 360 using the harmonic coefficients of the model EGM96 beyond the maximal degree 200 of GGM02C. The medium wavelength components were computed using the best gridded residual gravity anomalies, by integration in Stokes' formula. The digital terrain model GLOBE contributed to its short wavelength components. The residual terrain model (RTM) was applied to first determine a quasi-geoid model. This intermediate surface was converted to the geoid using a grid of simple Bouguer gravity anomalies. The validation of CGM05 is based on comparisons to global and regional geoids. A GPS/levelling geometric geoid computed in a small part of the target area shows that the absolute accuracy of this local geoid model is 14 cm. After a four-parameter fitting to the GPS/levelled reference surface, this absolute accuracy reduced to 11 cm.*

**Keywords:** *geoid, quasi-geoid, GPS/levelling, global geopotential model, gravity anomalies.*

## 1 Introduction

The geoid is the equipotential surface of the Earth's gravity field. The determination of this surface has not always attracted the attention of geodesists in Africa as in the developed world. However, one of the major problems of surveyors today is accurate height determination. Spirit levelling is expensive and time consuming. Therefore, the users prefer the modern Global Positioning System (GPS) method. This method gives positions of points in a terrestrial three-dimensional frame. The conversion of the GPS geometrical heights  $h$  into heights above the geoid needs the geoid-ellipsoid separation or geoid undulation. Orthometric heights  $H$  are deduced from geoid undulations  $N$  and heights above the reference ellipsoid by the following equation:

$$H = h - N \quad (1)$$

In addition to this direct practical function, the knowledge of the geoid is of scientific interest in the contribution it makes to the understanding of the earth's crustal structure. There are two main approaches in geoid determination in the central African subregion: the GPS/levelling and gravimetric options. This area is characterised by a poor spatial coverage of precisely levelled data points. In addition, interpolation/extrapolation would be necessary in mountainous areas, where the geoid is more variable. Therefore, the GPS/levelling geometric geoid is not an appropriate method. The gravimetric approach depends mainly on the better distribution, precision, density of gravity stations and the quality of the global geopotential model (GGM) used. This approach was used for the computation of the local geoid of Cameroon, CGM05 (Cameroonian Geoid Model 2005). Gravity data used contained terrestrial and marine data, combined with satellite altimetry data. They were evaluated and validated in order to detect gross errors. Using the residual terrain model method (Forsberg, 1984) in conjunction with the remove-restore technique, a model of quasi-geoid was first computed. This surface was further converted into the desired geoid, using a grid of simple Bouguer anomalies. A hybrid GGM (EGM-GGM) supplies the longer wavelength components of the gravity data and the final geoid. This GGM is obtained by extending the GGM02C (Tapley et al., 2005) to degree and order 360 using the harmonic coefficients of EGM96 (Lemoine et al., 1998) beyond the maximal degree 200 of GGM02C. The medium wavelength components were computed from a 5'×5' grid of residual gravity anomalies after integration in Stokes's formula. The digital terrain model (DTM) GLOBE (Hasting and Dunbar, 1998) contributed with the shortest wavelengths.

## 2 Data Acquisition and Preparation

### 2.1 Gravity data

Gravity data contained 62,000 points, mainly owned by the Institut de Recherche pour le Developpement IRD (France) and released by the Bureau Gravimétrique International (BGI). These data originating from different sources, were combined

with those from satellite altimetry (Sandwell et al., 1995). The extent of the data area (Fig. 2) covers a larger area than the geoid solution itself (Fig. 1), in order to minimise the edge effects. A simplified strategy allowed the detection of gross errors in these data. The strategy accounts for:

- Points with no data value;
- Stations with a height greater than the neighbouring ones and than the value interpolated in the DTM;
- Marine gravity data which appears to be on land and vice versa;
- Stations with alphanumeric characters instead of numeric;
- Duplicated gravity data and gravity data much greater than the nearest values.

The gravity anomalies of the BGI database were recalculated. The results show that this database is not homogeneous. There was a difference of  $-18$  mGal appearing at some stations. This difference is probably due to the difference which exists between IRD gravity base stations established in Africa (Duclaux et al., 1954) and the Potsdam base station. These data were corrected by the following equation (Levallois, 1977):

$$g = g_{BGI} - 17.696 + 0.001227(g_{BGI} - 978500) \quad (2)$$

where  $g$  (mGal) is the corrected data and  $g_{BGI}$  (mGal) the data from the BGI database. In marine areas, some gravity values were corrected considering the Potsdam base value by the following equation (Li and Götze, 2001):

$$\delta g = -16.3 + 13.7 \sin^2 \phi \quad (3)$$

where  $\delta g$  is the correction to add (in mGal) and  $\phi$  the latitude of the station.

Gravity anomalies were computed using modern formula, as described by Blakely (1996) and Moritz (1980). The IRD data were tied to the “Martin network” of ORSTOM (Duclaux et al., 1954). The gravity stations of this network were determined with an accuracy of about 0.05 mGal (Poudjom Djomani et al., 1995). In this study, the error estimate for gravity anomalies on land is better than 2 mGal compared to 10 mGal in marine areas. This poor marine data accuracy is probably due to the instability of the platforms used.

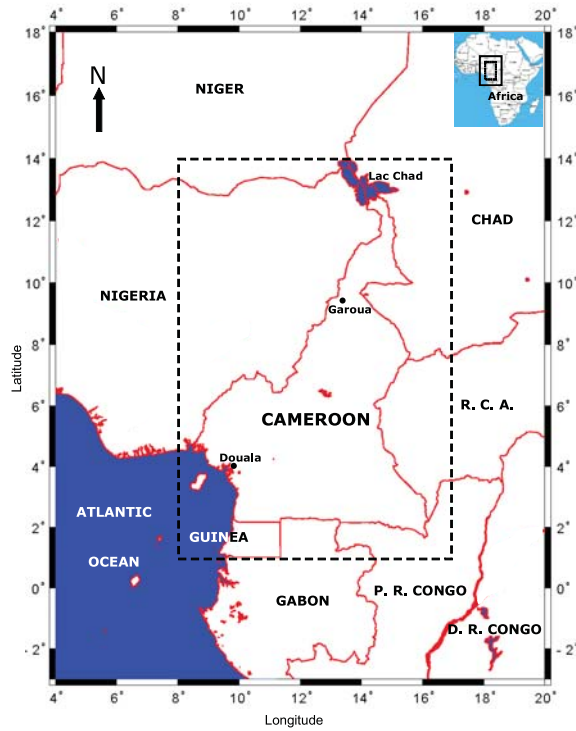


Figure 1. Data area and target area (in dotted line).

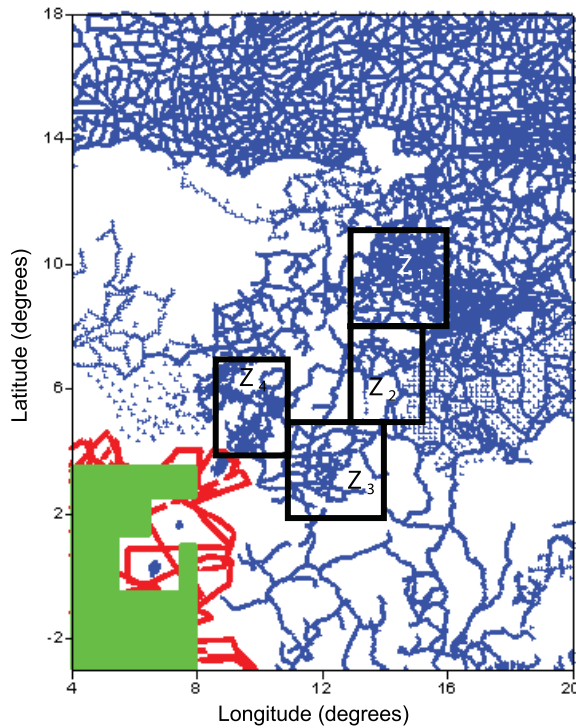


Figure 2. Gravity data coverage and test areas  
 + land data;  $\Delta$  Marine data;  $\bullet$  Satellite altimetry data.

## 2.2 GPS/levelling data

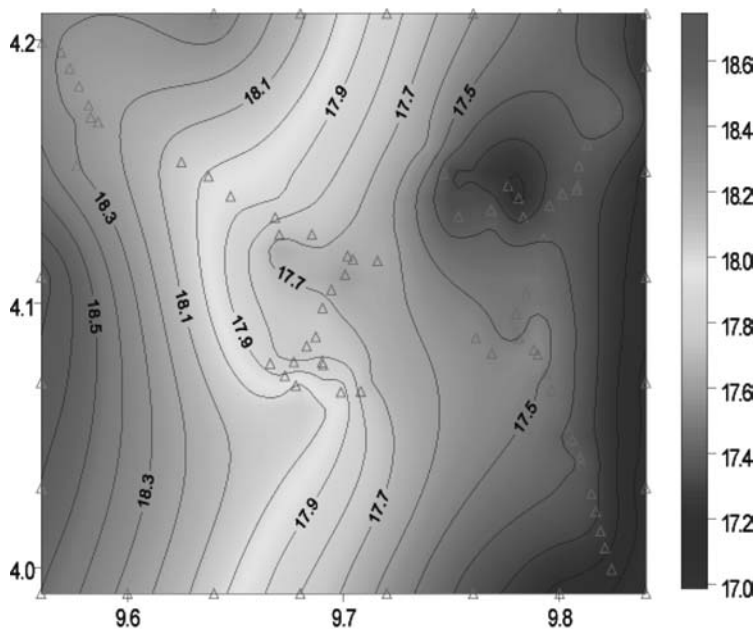
The geometric geoid obtained from GPS/levelling points is an independent data source used to evaluate a gravimetric geoid. The GPS network used here was established in 2000 by IGN-France. It consists of 250 stations, with 87 precisely levelled (Gueguen, 2001). The ellipsoidal and orthometric heights of these stations are used to compute the geometric geoid (equation 1), in a small area between latitudes  $3.94^{\circ}N \leq \phi \leq 4.17^{\circ}N$  and longitudes  $9.56^{\circ}E \leq \lambda \leq 9.85^{\circ}E$  (Fig. 3). The values of  $N$  obtained (Table 1) are also used to determine the more appropriate GGM for geoid computation in Cameroon and to evaluate the quality of the final geoid, CGM05.

**Table 1.** GPS/levelling data in the Douala region-Cameroon (unit: metre).

Number of stations	Max.	Min.	Mean	STD
87	18.74	16.98	17.66	0.41

## 2.3 The global geopotential model (GGM)

The harmonic coefficients of GGMs are used in gravimetric geoid determinations based on the remove-restore technique. The GGM chosen should best represent the geoid undulations and the gravity anomalies in the study area. EGM96 (Lemoine et al., 1998) is the most widely used. However, Merry (2003) concluded that this model does not represent precisely the gravity data of Africa. This author also showed that there is a significant difference between the medium order (to degree 90) geoid deduced from EGM96 and a preliminary GRACE model (GGM01) for Southern Africa. The GRACE models may therefore be suitable for geoid solutions in Africa.



**Figure 3.** GPS geometric geoid of Douala (Unit = metre ;  $\Delta$  = GPS/levelling points).

In order to select a GGM for Cameroon, four GGMs were tested: EGM96, OSU91A, EGM-GGM and GGM02C. The latest is a combined GGM developed to degree and order 200 (Tapley et al., 2005). Its coefficients result from a weighted combination of those of EGM96 and GGM02S. GGM02S is a satellite GGM developed to degree and order 160, deduced from fourteen monthly gravity field solutions (Tapley et al., 2005).

Gravity anomalies ( $\Delta g_{GGM}$ ) computed from these GGMs using equation (4) are compared to land, marine and satellite altimetry anomalies. The geoid undulations ( $N_{GGM}$ ) of the GGMs (equation 5) are compared to the geometric geoid undulations computed in the Douala region.

$$\Delta g_{GGM} = \frac{GM}{r^2} \sum_{n=2}^{N_{max}} \left(\frac{a}{r}\right)^n (n-1) \sum_{m=0}^n (\Delta \bar{C}_{nm} \cos m\lambda + \Delta \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\sin\phi) \quad (4)$$

$$N_{GGM} = \frac{GM}{\gamma r} \sum_{n=2}^{N_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (\Delta \bar{C}_{nm} \cos m\lambda + \Delta \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\sin\phi) \quad (5)$$

In equations 4 and 5,  $GM$  is the geocentric gravitational constant;  $(r, \lambda, \phi)$  are the spherical coordinates of the computation point;  $\gamma$  is the normal gravity on the reference ellipsoid;  $a$  is the equatorial radius of the earth;  $\bar{P}_{nm}(\sin\phi)$  are the fully normalized associated Legendre functions for degree  $n$  and order  $m$ ;  $\Delta \bar{C}_{nm}$  and  $\Delta \bar{S}_{nm}$  are the normalized EGM-GGM coefficients, reduced for even zonal harmonics for the ellipsoid.

The following analysis is based on the standard deviations (STD) of the differences between anomalies and between undulations. A GGM is more representative of the data in the study area if the corresponding STD is small. The following tables (2–6) show the results of comparisons.

**Table 2.** Differences between terrestrial gravity anomalies and GGM anomalies (unit: mGal; 36,427 points used).

GGM	Max.	Min.	Mean	STD
<b>CGM02C</b>	275.96	-89.49	0.40	21.07
<b>EGM96</b>	245.21	-86.97	4.47	17.05
<b>OSU91A</b>	248.85	-101.91	0.30	17.78
<b>EGM-GGM</b>	243.08	-84.24	-0.13	16.80

**Table 3.** Differences between marine gravity anomalies and GGM anomalies (unit: mGal; 6,389 points used).

GGM	Max.	Min.	Mean	STD
<b>CGM02C</b>	158.55	-83.77	-9.82	20.04
<b>EGM96</b>	149.37	-83.01	-8.75	16.97
<b>OSU91A</b>	158.68	-74.80	-7.84	16.97
<b>EGM-GGM</b>	148.92	-82.20	-8.97	17.20

**Table 4.** Differences between satellite altimetry anomalies and GGM anomalies (unit: mGal; 18,615 points used).

<b>CGM</b>	<b>Max.</b>	<b>Min.</b>	<b>Mean</b>	<b>STD</b>
<b>CGM02C</b>	178.01	-42.64	0.52	12.48
<b>EGM96</b>	161.97	-36.29	0.21	10.64
<b>OSU91A</b>	163.54	-37.18	0.20	10.70
<b>EGM-GGM</b>	162.03	-36.00	7.45	10.60

**Table 5.** Differences between the geometric geoid and GGM undulations (unit: metre; 87 points used).

<b>GGM</b>	<b>Max.</b>	<b>Min.</b>	<b>Mean</b>	<b>STD</b>
<b>GGM02C</b>	0.75	0.00	0.35	0.28
<b>EGM96</b>	0.68	-0.26	0.28	0.22
<b>OSU91A</b>	1.17	0.52	0.82	0.29
<b>EGM-GGM</b>	0.92	0.04	0.53	0.20

**Table 6.** Statistics of GGM undulations in the target area (unit: metre; 61,431 points used).

<b>GGM</b>	<b>Max.</b>	<b>Min.</b>	<b>Mean</b>	<b>STD</b>
<b>GGM02C</b>	26.66	-17.00	13.52	6.51
<b>EGM96</b>	26.73	-17.31	13.50	6.60
<b>OSU91A</b>	25.43	-16.89	13.32	6.79
<b>EGM-GGM</b>	26.45	-17.01	13.53	6.51

From tables 2–6, the smallest STD is always obtained with model EGM-GGM, except for the marine anomalies (Table 3). This model is chosen to support the long wavelength components of gravity anomalies and geoid undulations in Cameroon. EGM-GGM nearly behaves like EGM96. The reason may be that no other gravity data of Africa were added to those used for EGM96 during the computation of GGM02C. Marine data do not agree with all the GGMs (Table 3). The mean bias is about -9 mGal for the marine anomalies (Table 3) and practically less than 1 mGal for the others. The main reason for this is probably the poor accuracy of these gravity data. Their accuracy is about 10 mGal, against 2 mGal for land data as indicated in the BGI database and 3 to 6 mGal for satellite altimetry gravity anomalies (Arabelos and Tziavos, 1994; Sandwell and Smith, 1997).

#### **2.4 The digital terrain model (DTM)**

The African continent is poorly covered by local digital terrain models (Merry, 2003) and Cameroon is not an exception. The global DTM GLOBE was used instead. GLOBE was also used in the computation of the African Geoid Project APG2003 (Merry, 2003). The author concluded that GLOBE was poorly representative of the external topography in Africa. However, when CGM05 was computed, the recent SRTM, (NASA press release, 2003) was being tested. This model will soon be used, when the Cameroonian geoid will be updated.

**Table 7.** Differences between heights from GLOBE and those of the BGI database (units: metre).

Source	Number	Max.	Min.	Mean	STD
<b>BGI database</b>	44,000	2,258.00	0.00	399.61	313.34
<b>GLOBE</b>	5,961,600	4,059.00	0.00	363.57	255.09
<b>Differences</b>	44,000	1,607.54	-60.83	312.22	166.76

GLOBE is represented here on a grid of  $1 \times 1 \text{ km}^2$ , in an area extending beyond the data area (Fig. 2), from latitude  $-1^\circ\text{N}$  to  $19^\circ\text{N}$  and from longitude  $3^\circ\text{E}$  to  $21^\circ\text{E}$ . The maximum height here is 4,059 m. One can note that the highest summit of the study area, the Mount Cameroon, measures about 4,100 m. Therefore, the GLOBE grid interval is too large to precisely represent the topography in the region. Nevertheless, by considering the maximum height and the STD in table 7, heights obtained from GLOBE were considered more accurate than those of the BGI database and so the GLOBE digital elevation model was used. From table 7, one can conclude that there is no gravity station on Mount Cameroon.

### 3 Practical Computation of CGM05

#### 3.1 Theory of the gravimetric geoid determination

The basis for the geoid determination is the so-called “remove-restore” technique (Forsberg and Tscherning, 1981). The geoidal height ( $N$ ) can be derived from the height anomaly ( $\zeta$ ), the quasi-geoid/ellipsoid separation, using the expression (Rapp, 1997):

$$N = \zeta + \frac{\Delta g_B}{\bar{\gamma}} H \quad (6)$$

where  $\bar{\gamma}$  is the mean value of the normal gravity computed along the plumbline,  $H$  the orthometric height and  $\Delta g_B$  is the simple Bouguer anomaly. The height anomaly ( $\zeta$ ) at each point is considered as formed by a long wavelength part ( $\zeta_{CGM}$ ); a medium wavelength component, the residual height anomaly ( $\zeta_{res}$ ) and a short wavelength part ( $\zeta_{RT}$ ), so that:

$$\zeta = \zeta_{CGM} + \zeta_{res} + \zeta_{RT} \quad (7)$$

( $\zeta_{CGM}$ ) is obtained from the harmonic expansion of a GGM. The residual height anomaly ( $\zeta_{res}$ ) is obtained from integration of interpolated residual anomalies in Stokes’s formula. The short wavelength part ( $\zeta_{RT}$ ), which is computed using a DTM, is the residual terrain effect on the height anomaly. It is given by Duquenne (2005):

$$\zeta_{RT} = \frac{GR^2}{\gamma} \iint_{\sigma} \frac{\rho(H - H_0)}{r} d\sigma \quad (8)$$

where  $\gamma$  is the normal potential,  $R$  is the mean radius of the earth,  $G$  the earth gravitational constant,  $\rho$  the terrain rock density,  $H$  the height of the roving point,



$H_0$  the height of the computation point,  $r$  the distance between the computation and the roving points, and  $d\sigma$  a surface element of the unit sphere  $\sigma$ .

The way gravity data are processed introduces a correction term called the indirect effect on gravity and height anomalies. This indirect effect is due to the constraints related to the application of Stokes's formula (Heiskanen and Moritz, 1967). Many types of topographic corrections can be used to meet some of the above constraints. The Bouguer correction always approximates the topographic masses to a plateau. Depending on the degree of accuracy needed, a terrain correction can be added in order to account for real topographic shape (Schwarz et al., 1990). The isostatic correction is used to regularize the crust according to a model of compensation. Pratt and Airy models are generally adopted in this correction. The Helmert condensation supposes all the topographic masses condensed to form a layer on the geoid (Heiskanen and Moritz, 1967). This probably reduces the distortions introduced by other topographic corrections and reduces the indirect effect (Duquenne, 2005). This indirect effect is one of the criteria that can be used to value the merit of a topographic correction. The less the indirect effect, the better the method (Heiskanen and Moritz, 1967).

The RTM method considers that the real topography consists of two parts: the filtered part which best represents the variations of the GGM and the residual part not accounted for by the GGM. The latest is the difference between the real and the filtered terrain. The corrections considered concern the residual terrain and are small quantities. The indirect effect  $\Delta g_{ind}$  related to gravity data reduction is given by (Forsberg, 1984):

$$\Delta g_{ind} = \frac{1}{\gamma} \frac{\partial \gamma}{\partial h} T_{RT} \quad (9)$$

Where  $\gamma$  is the normal gravity,  $\partial \gamma / \partial h$  is the normal gravity gradient and  $T_{RT}$  the anomalous potential due to the residual terrain. The RTM method helps remove the GGM components and the terrain effects from the gravity data at the same time. The secondary indirect effect induced by this method is a small quantity (Forsberg, 1984). In this study, this quantity was neglected in the gravity reductions.

### 3.2 Interpolation of gravity data

In gravimetric geoid determination, the best interpolation method should be used, along with the appropriate type of gravity anomaly. Five types of gravity anomalies and four interpolation methods were tested in this study. These methods are: the Minimum Curvature Splines in Tension (Smith and Wessel, 1990); the Least Square Polynomial Fitting; simple Kriging (Krige, 1978) and the inverse distance to a power method. The best method was selected on the criterion of Crain and Bhattacharya (1967) by which at a given location, the measured data should be close to the data interpolated from the neighbouring values. In order to manage the data without losing information, a  $5' \times 5'$  grid was considered. Four test areas  $Z_1$ ,  $Z_2$ ,  $Z_3$  and  $Z_4$  of different characteristics were chosen (Fig. 2) in order to conduct the analysis. Their selection was based on the density and the geographical distribution of the gravity net. The relative roughness of the topography relief and

the relative complexity of the geological units encountered were also considered. The test area  $Z_1$  (Fig. 2) is located in the northern sedimentary region of Cameroon.  $Z_2$  covers part of the Adamawa uplift in central Cameroon.  $Z_3$  is located on the continental extension of the Cameroon Volcanic Line.  $Z_4$  lies on the northern edge of the Congo Craton.

A statistical comparison was made in each test area between gravity anomalies predicted directly and those obtained from: (a) predicted simple Bouguer anomalies; (b) predicted refined Bouguer anomalies; (c) predicted residual anomalies, each previously transformed into predicted total anomalies. The residual anomalies were computed using the best GGM in the study area. Since comparison must be made between total gravity anomalies, each type of gravity anomaly was introduced in a form of remove-interpolation-restore technique. The STD of the differences was computed in each test area. The smallest STD was obtained with the residual anomalies and the kriging method in  $Z_1$ ,  $Z_3$  and  $Z_4$ . This method was therefore used at each stage of interpolation with the residual anomalies. Kriging is a general technique that can be tuned and implemented in many different ways. In this work, only simple kriging was tested.

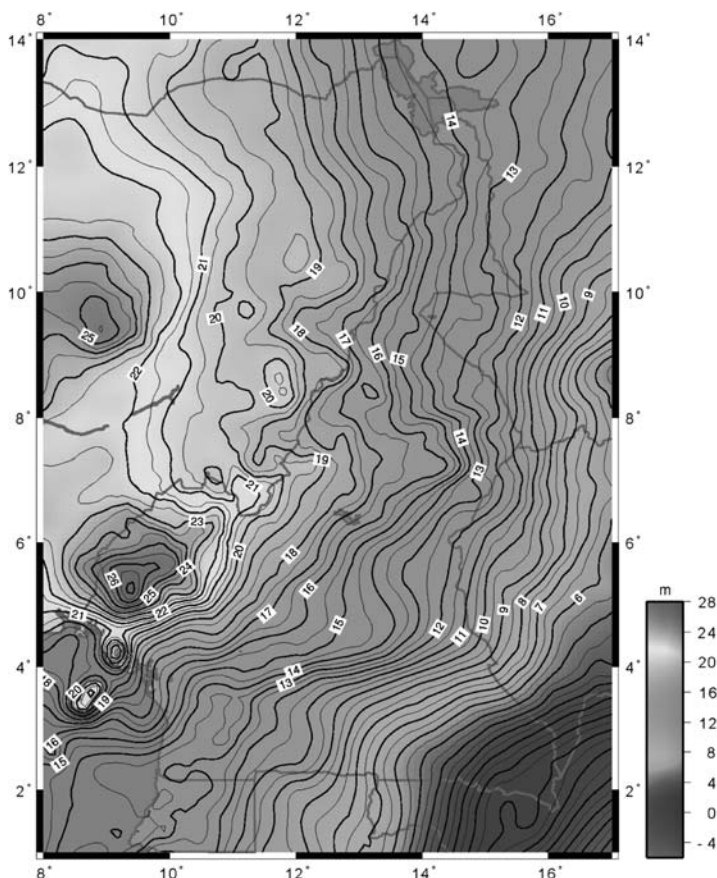
### **3.3 Computation of CGM05**

The computation of different parts of CGM05 is done using the GRAVSOFIT package (Tscherning et al., 1991; 1992) and other complementary algorithms. The geoid grid has the same interval as the interpolated gravity anomalies in the study area. These gravity anomalies are plotted on figure 5 for the target area. The GRAVSOFIT package contains many software. The programs TCGRID and TC were used respectively to produce two DTMs from the detailed GLOBE and to compute the terrain corrections on gravity anomalies respectively. The mean DTM is obtained from the detailed GLOBE by simple averaging. The coarse DTM is used in the outer zone around each computation point. The contribution of EGM-GGM to the gravity anomalies is computed and combined to the terrain corrections on gravity anomalies. The subtraction from the gravity anomalies computed on actual earth surface yields the residual anomalies. These residual anomalies were interpolated on a regular grid and integrated in Stokes's formula with no kernel modification, using the program STOKES. The use of Stokes's formula with no kernel modification can affect the quality of the final geoid. One of the consequences is that long-wavelength errors in the gravity anomalies are allowed to freely affect the computed geoid heights. Many modifications to this kernel can be used (Wong and Gore, 1969; Vincent and March, 1974). These methods are developed under the assumption that the errors stemming from the harmonic coefficients and measured gravity anomalies are negligible. The gravity net of Cameroon is being densified and when the geoid model will be updated, the above modifications will be tested, along with others such as the least squares modification (Sjoberg, 1991).

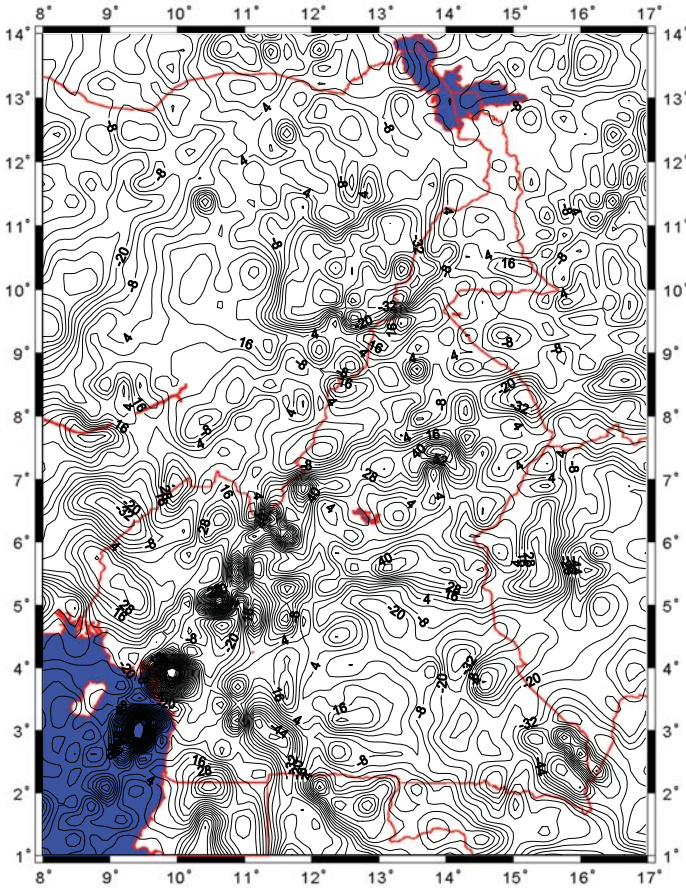
The residual height anomalies obtained are further combined to the residual terrain effect on height anomaly. The latest are computed using the program TC and the three DTMs. Finally, the long wavelength component of the height anomaly

is computed using the harmonic coefficients of EGM-GGM. The combination to the residual height anomaly and the residual terrain effect yields the quasi-geoid height anomaly.

The final geoid is shown on figure 4. The maximum undulation is 27.16 m while the minimum is  $-5.58$  m, with a mean value of 14.87 m and a standard deviation of 6.11 m. The negative values observed in the south-east indicate that CGM05 undulates below the reference ellipsoid. Two geoid highs (A and B) on the CGM05 map (Fig. 4) are located in the northwest of the Benue Trough in Nigeria and in the east of the Mamfe basin in Cameroon. These highs probably mark the presence of high-density rocks (Kamguia et al., 2007). These materials have already been identified in the Benue Trough (Cratchley et al., 1984; Benkelil, 1988; Ofoegbu and Mohan, 1990). This study has revealed the presence of those near the Mamfe basin. These two geoid highs were also observed on the regional geoid AGP2003 contour map (Merry et al., 2003), computed for the African continent.



**Figure 4.** The local geoid model of Cameroon. Two geoid highs are located in the northwestern end of the Benue Trough in Nigeria (A) and in the east of the Mamfe Basin in Cameroon (B). (Mercator projection, Interval: 0.5 m).



**Figure 5.** Gravity anomaly map of Cameroon (Mercator projection, Interval : 6 mGal).

#### 4 Comparison to the GPS/Levelling Geoid

The geoid CGM05 was compared to global and regional geoid models as well as a geometric GPS geoid. The global models are GGM02C, EGM96, OSU91A and EGM-GGM. The regional geoid is a  $5' \times 5'$  grid of AGP2003 over Cameroon. The geometric geoid is shown on figure 3. The validation is first based on the statistical comparison between the above global and regional geoids (Table 8).

From table 8, EGM-GGM and EGM96 have the smallest STD. Meanwhile, CGM05 has a smaller STD than AGP2003. When comparing the differences of undulations, the smallest STD is obtained with EGM-GGM and EGM96. The maximum value of the difference between CGM05 and AGP2003 is 2.19 m. These differences can be attributed to GGMs used during their computations, the gravity datasets considered in marine areas for each model and differences in the computational approaches. Moreover, some gaps appearing in the land data of figure 2 were filled during the computation of AGP2003 with 2,500 data.

These data were not available during the computation of CGM05. The STD of the differences for the two models is 52 cm.

In order to obtain the absolute accuracy of CGM05, a geometric geoid was used (Fig. 3). For each of the 87 points, the undulation  $N_{GPS/lev}$  is known and the CGM05 undulation  $N_{CGM05}$  is interpolated in the grid. 87 differences  $\delta N = N_{GPS/lev} - N_{CGM05}$  are then computed. The undulations of these points from three other models were also computed and their differences with  $N_{GPS/lev}$  determined (Table 9). From this table, the smallest STD (14 cm) is obtained when comparing CGM05 to the geometric geoid. The STD for the other models is 17 cm.

**Table 8.** Statistics of models (16,848 points) and the differences of their undulations around Douala (87 points).

Models	Max.	Min.	Mean	STD
<b>Statistics of global and regional geoid models</b>				
<b>EGM96</b>	23.30	-6.40	14.21	5.93
<b>AGP2003</b>	27.18	-5.77	14.51	6.29
<b>CGM05</b>	27.16	-5.58	14.87	6.11
<b>EGM-GGM</b>	25.08	-6.44	14.76	5.88
<b>Statistics of differences between models</b>				
<b>EGM-GGM – EGM96</b>	2.77	-1.07	0.55	0.43
<b>AGP2003 – EGM96</b>	5.87	-1.92	0.30	0.81
<b>AGP2003 – EGM-GGM</b>	5.97	-2.60	-0.25	0.89
<b>CGM05 – EGM96</b>	5.96	-1.01	0.66	0.70
<b>CGM05 – EGM-GGM</b>	6.16	-1.67	0.10	0.70
<b>CGM05 – AGP2003</b>	1.66	-2.19	0.31	0.52

**Table 9.** Comparison of local and global geoids to the geometric geoid of Douala (Unit: metre, 87 points).

Differences	Max.	Min.	Mean	STD
$N_{GPS/lev} - N_{EGM96}$	0.70	-0.31	0.28	0.17
$N_{GPS/lev} - N_{APG2003}$	0.44	-0.32	0.02	0.17
$N_{GPS/lev} - N_{EGM-GGM}$	0.92	0.04	0.53	0.17
$N_{GPS/lev} - N_{CGM05}$	0.70	0.06	0.39	0.14

**Table 10.** Comparison of CGM05 to the geometric geoid before and after adjustment (Unit: metre, 87 points).

Differences	Max.	Min.	Mean	STD
$N_{GPS/lev}^i - N_{CGM05}^i$ (Before)	0.70	0.06	0.39	0.14
$N_{GPS/lev}^i - N_{CGM05}^i$ (After)	0.22	0.00	0.08	0.11

The difference between  $N_{GPS/lev}$  and  $N_{CGM05}$  in the region of Douala is not nil. One can conclude that the reference surface of orthometric heights obtained from GPS/levelling does not coincide with the surface deduced from gravity data.

CGM05 will be used to determine orthometric heights from GPS in Cameroon. It should be fitted to the reference surface obtained from GPS/levelling. This offers another advantage that the adjusted geoid model now serves as the unique reference for height determination in the area. For a GPS/levelled point  $i$  with coordinates  $(\phi_i, \lambda_i)$ , the difference  $\delta N^i = N_{GPS/lev}^i - N_{CGM05}^i$  was modelled and computed by a regional tendency (Forsberg and Madsen, 1990):

$$\delta N^i = N_{GPS/lev}^i - N_{CGM05}^i = h_i - (H_i + N_i) = a + b \cos(\phi_i - \phi_0) \cos(\lambda_i - \lambda_0) + c \cos(\phi_i - \phi_0) \sin(\lambda_i - \lambda_0) + d \sin(\phi_i - \phi_0) \quad (10)$$

where  $(\phi_0, \lambda_0)$  is the mean position in the local geoid (Fig. 3).

The adjustment consists of determining the four parameters  $a$ ,  $b$ ,  $c$  and  $d$  of equation (10) by the least squares method. The geoid CGM05 in Douala was further corrected by  $\delta N^i$  at each point  $i$  and compared to the GPS/levelling geoid. The results are shown in table 10. The STD of the differences after the fitting is 11 cm, which means an amelioration of 22%. This amelioration is small, probably due to errors related to the spirit levelling in Cameroon. The STD of 11 cm can be considered as the accuracy of orthometric heights determined from GPS measurements in Douala using the gravimetric geoid of Cameroon, CGM05.

## 5 Conclusion

The local geoid of Cameroon has been computed, from gravity data. This geoid will be introduced in the height determination in Cameroon. Its undulations decrease from the western part of the area to the East (Fig. 4). The analysis of this decrease and other geoid features may be useful in geophysics. This analysis can constrain the limiting depths of some geological features of the Cameroon subsurface, very useful in the gravity and magnetic modeling of the subsurface. It can also lead to the precise mapping of the positions of some tectonic features and completion of their geological and geophysical studies using independent data and method.

The negative values observed in the south-east indicate that CGM05 undulates below the reference ellipsoid. There are two geoid highs on the geoid map, located east of the Mamfe basin and in the upper Benue Trough in Nigeria. Through a comparison with the GPS/levelling derived geometric geoid in a small part of the study area, its absolute accuracies are 14 cm and 11 cm, respectively before and after a four-parameter fitting to the GPS/levelled reference surface. This accuracy is better than the one obtained with AGP2003 in Cameroon.

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